Carbon Taxation When Climate Affects Productivity

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ABSTRACT. *Based on a model where climate change affects productivity, the second-best optimal carbon tax is found to exceed marginal social damage by 53% and ''marginal private damage'' (aggregate households' willingness to pay) by 73%. Annual welfare gains are estimated at $3.58 billion when marginal damage is $40 per ton; employment also increases. A carbon tax set at the Pigouvian rate raises welfare by only $3.17 billion. The seemingly contradictory results from the ''tax interaction'' literature are shown to arise only when the optimal environmental tax is compared to ''marginal private damage,'' and only for an amenity externality.* (JEL Q4, H2*)*

I. INTRODUCTION

Global climate change is thought to be among the most serious environmental threats to humanity, one that presents enormous intellectual and policy challenges in a complex setting where the stakes are high. The policy questions surrounding climate change have attained heightened importance since 1997, when 160 nations signed the Kyoto Protocol and agreed to significantly reduce emissions of greenhouse gases. This situation represents an important opportunity for economists to provide policymakers with sound analysis and clear guidance at a time when there is growing recognition of the advantages of market-based policies. In this context, there has been renewed interest and considerable controversy surrounding optimal environmental taxation in a second-best setting with preexisting taxes.

Tullock (1967) was the first to suggest that revenues from environmental taxes could be used to finance reductions in preexisting revenue-motivated taxes as a way to improve the environment and reduce the welfare costs associated with the overall tax program. This notion, now widely referred to as the ''double dividend hypothesis,'' also carries with it the intuitive inference that the optimal environmental tax would generally exceed marginal environmental damages when the revenues are used in this way (see Terkla 1984; Lee and Misiolek 1986; Pearce 1991; and Oates 1995).

Despite the intuitive appeal of the double dividend hypothesis, a more recent literature has raised questions about its validity and its implications for setting optimal environmental taxes. Although the authors of this literature concur that the use of environmental tax receipts to finance reductions in preexisting taxes will indeed have a positive welfare effect, they find that the optimal environmental tax lies below, rather than above, the marginal social damage from pollution even when environmental tax revenues are used to finance reductions in preexisting taxes (Bovenberg and de Mooij 1994; Parry 1995; Bovenberg and Goulder 1996). To explain this result, the authors postulate that a previously unrecognized welfare cost or ''tax interaction effect'' offsets the positive effect associated with the double dividend. The unexpected implication of these results and their interpretation is that the welfare gains from environmental tax reform, and the ability to control pollution efficiently, is much more limited than previously thought.[1] If correct, these results are believed to have profound implications not just for climate change policy but for many other environmental and non-environmental policies as well (Parry and Oates 2000).

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[1] This now-extensive literature also includes Goulder (1995), Fullerton (1997), Schöb (1997), Bovenberg and de Mooij (1997), Goulder, Parry, Burtraw (1997), Parry, Williams, Goulder (1999), Goulder et al. (1999), Parry and Bento (2000), among others.

The approach taken in much of this recent tax interaction literature has been to specify analytical or numerical general equilibrium models where an environmental amenity affects utility directly, where labor is the only productive factor and source of income, and where demands for polluting and non-polluting goods are assumed to be similar (so that in the absence of an externality, uniform taxation would be optimal to raise revenues).

The current analysis employs a similar model, but one in which climate change damage reduces welfare by lowering factor productivity, rather than as an amenity externality. For a numerical model of the U.S. economy, a welfare-maximizing algorithm is employed to choose optimal taxes which both raise revenues and control carbon emissions. The analysis finds that the optimal carbon tax exceeds marginal social damages from climate change by about 53%, a result consistent with the double dividend hypothesis. These results are also shown to be in harmony with analytically derived expressions for the optimal environmental tax.

A comparison of the current analysis with that from the tax interaction literature reveals that two different definitions of marginal social damage are being used. In the current analysis, the definition of marginal social damage is based on the social marginal rate of substitution between income and environmental quality, so that the social marginal utility of income is the numeraire unit of value. By contrast, the tax interaction literature uses a definition based on the marginal private damage for an individual household, and then summed over the population. The two definitions will differ whenever there are revenue implications from changes in either income or climate, except at the first-best optimum. When the definition used in the tax interaction literature is applied to the current model, the optimal carbon tax exceeds marginal social damages by an even greater amount, 73%. Indeed, only for the case of an amenity externality, *and* when marginal damage is defined as the sum of households' marginal private damage, does the relationship between the optimal carbon tax and marginal damage appear to be incongruous with the double dividend hypothesis.

The general model and numerical simulation results are presented in Section 2. Section 4 evaluates the differences with the tax interaction literature using both the numerical model and analytical results. Section 4 concludes.

II. THE MODEL

We begin with a climate economy model which contains two endowments, income in the form of household time (T), and a global climate. There are m identical households who allocate their time between leisure (V) and labor supply (L), so that $T - L = V$. Utility is a function of consumption (C) and leisure. Consumer goods are produced with two intermediate inputs, one using fossil fuels (F), and one using non-carbon inputs (N), such that $C = c(F, N)$. We can therefore write utility as

$$U = u(C, (T - L)). \quad [1]$$

Production is assumed to be competitive, and labor is assumed to be the only input used to produce the intermediate inputs F and N.

The model is assumed to be for one period and thus does not attempt to solve an explicit dynamic problem. Expected carbon damages are therefore being characterized as contemporaneous damages rather than future consequences of cumulative carbon emissions. The structure of this model is similar to the one in Parry, Williams, and Goulder (1999).

The climate, E, is altered when carbon is emitted in fixed proportions to the consumption of F, such that $E = m\phi F$. A change in the climate is assumed to harm the economy due to its adverse effect on factor productivity. Many types of expected climate damages involve reduced output or losses of productivity, such as losses in the productivity of agriculture and forestry, losses of productive assets due to sea level rise, reduced productivity of fixed assets and infrastructure, the redirection of productive factors toward defensive and restorative efforts, or direct reductions in labor productivity from morbidity and mortality. Indeed, Cline (1992) estimates that about three-fourths of climate change damages will come from losses to agriculture, forestry, and damage from sea level rise.

For simplicity, and because labor is the only productive factor in the model, we characterize climate damage in terms of its affect on output per unit of labor. Specifically, labor productivity is defined as a function of carbon emissions, or $h(E)$, where $dh/dE < 0$. For a given E, the marginal product of labor is assumed to be constant so that output is a function of labor supply and climate.

Exogenously determined revenues, mG, are collected using excise taxes on F and N. These revenues are returned to households in the form of lump-sum transfers, rather than introducing an explicit and detailed representation of the public sector. Taxes can be interpreted as payments out of labor income. A tax on F is also a tax on carbon emissions given the fixed proportions relationship being assumed. We define output using a linear production function such that for an economy consisting of m households, aggregate output is $mh(E)L = mF + mN + mG$. Units are chosen so that marginal rates of transformation among F, N, and G all equal one.

Assuming competitive production and with no direct taxes on C, the tax problem will be unchanged if we substitute the production function into the utility function to reflect derived demands for F and N. We can therefore write the representative household's maximization problem as

$$\begin{aligned} &\underset{F,N,V}{\text{Max:}} \quad u(c(F,N),V) \\ &s.t.(\lambda) \quad (1+t_F)F + (1+t_N)N = h(E)L + G, \qquad [2] \end{aligned}$$

where λ denotes the private marginal utility of income corresponding to the value of the implied Lagrange multiplier.

To maximize social welfare, W, defined as the sum of individuals' utility, the planner's problem can now be written as

$$\begin{aligned} &\underset{t_F,t_N}{\text{Max:}} \quad m\left[\begin{array}{ll} \underset{F,N,V}{\text{Max:}} & u(c(F,N),V) \\ s.t.(\lambda) & (1+t_F)F + (1+t_N)N = \\ & h(E)L + G \end{array}\right] \\ &s.t.(\mu) \qquad mt_FF + mt_NN = mG. \qquad [3] \end{aligned}$$

Let μ denote the shadow value of a unit of public revenue corresponding to the shadow value of the implied Lagrange multiplier for the revenue constraint. For [3] we can express the social marginal utility of carbon emissions as π where

$$\pi \equiv \frac{dW}{dE} = m\lambda\frac{dh}{dE}L + m\mu\left(t_F\frac{\partial F}{\partial(hL)}\frac{\partial(hL)}{\partial E} + t_N\frac{\partial N}{\partial(hL)}\frac{\partial(hL)}{\partial E}\right). \qquad [4]$$

Each term includes m due to the non-rival characteristic of E. (In an explicitly dynamic model we would want to write this as the present discounted value of expected losses in income due to induced changes in global climate over n years). Note that this expression includes both the direct effect on utility from reductions in private consumption (the first term on the right-hand side), plus the indirect effect due to lost revenues (the remaining terms on the right-hand side). Let Y denote a unit of income corresponding to the household budget constraint where units of time, T, are paid a wage $h(E)$. The social marginal utility of a unit of income, α, can then be expressed as

$$\alpha \equiv \frac{\partial W}{\partial Y} = \lambda + \mu\left(t_F\frac{\partial F}{\partial Y} + t_N\frac{\partial N}{\partial Y}\right) + \pi m\phi\frac{\partial F}{\partial Y}. \qquad [5]$$

This expression reflects the standard definition of the social marginal utility of income from Diamond (1985), but with the addition of the environmental damage term: the final term on the right-hand side reflects the disutility from additional carbon damages arising as a function of the marginal propensity to consume fossil fuels where $dE = m\phi dF$. The marginal social damage (MSD) from consumption of F is thus defined as the product of marginal emissions, $m\phi$, and the marginal rate of substitution (MRS) between income and emissions, or

$$\text{MSD} = \text{MRS}\frac{dE}{dF} = \frac{\partial W/\partial E}{\partial W/\partial Y}\frac{dE}{dF} = \frac{\pi}{\alpha}m\phi. \qquad [6]$$

To examine the relationship between the optimal carbon tax and marginal social damages, we note that the optimal taxes on F and N will reflect both revenue-raising taxes and, in the case of F, an environmental component. In order to isolate the environmental component of the optimal tax on F, we will assume that both F and N are average substitutes for leisure—as has been the standard approach in the recent "tax interaction" literature. With appropriate restrictions on preferences, the optimal carbon tax can then be interpreted as $t_F^* - t_N^*$. The relationship between the optimal carbon tax and marginal social damage is estimated in the next section based on a numerical model for the U.S. economy.

III. NUMERICAL MODEL AND RESULTS

The numerical general-equilibrium model involves constant elasticity of substitution functions for utility, $u(C, V)$, and production, $c(F, N)$, and is represented as a single period model rather than as a dynamic optimization problem. This approach is similar to the model of the U.S. economy introduced in Parry, Williams, and Goulder (1999). Indeed, the structure and parameters of the current numerical model replicate their model, albeit with a more aggregated representation of production, but calibrated to produce a similar marginal abatement cost curve.[2] Additional details of the model's structure and calibration are presented in Appendix A.

For purposes of numerical estimation, nested optimization models such as [3] are often represented as a single-maximization problem by introducing the household's first-order conditions as constraints on social maximization, similar to the first-order approach in principle-agent problems (Jewitt 1988). Letting subscripts denote partial derivatives with respect to variable j (e.g., U_j and C_j), we write the social welfare maximization problem as

$$\max_{t_F, t_N} \; m[u(C(F,N), V)]$$

$$\begin{aligned} s.t.\quad (\alpha)\quad & (1+t_F)F + (1+t_N)N = h(E)L + G \\ (\mu)\quad & t_F F + t_N N = G \\ (\eta_1)\quad & U_C C_F(1+t_N) = U_C C_N(1+t_F) \\ (\eta_2)\quad & U_V(1+t_F) = U_C C_F h(E) \\ (\pi)\quad & E = m\phi F. \end{aligned} \qquad [7]$$

TABLE 1

OPTIMAL CARBON TAXATION: RESULTS FOR NUMERICAL MODEL OF THE UNITED STATES (MARGINAL SOCIAL DAMAGE (MSD) = $40 PER TON)

Optimal tax, F	0.779
Optimal tax, N	0.641
Social marginal utility of emissions (π)	−15.98
Social marginal utility of income (α)	0.400
Social marginal utility of revenues (μ)	0.429
Optimal carbon tax on $F(t_F^* - t_N^*)$	0.138
(per ton of carbon)	$61.3
Private marginal utility of income (λ)	0.255
Private marginal utility of emissions (π_P)	−9.03
Ratio of optimal carbon tax to MSD (π/α)	1.53
Ratio of optimal carbon tax to MPD(π_P/λ)	1.73

Note: Resource allocation at the optimum is: $N = 2{,}620{,}500$, $F = 556{,}040$, $C = 1{,}763{,}700$, $V = 931{,}080$, and $E = 1{,}251.726$.

In this model, the shadow value of the Lagrange multiplier on income, α, will reflect the social value of a unit of income because all optima in this model represent Pareto efficient states. For that reason, the private marginal utility of income, λ, does not appear directly in the model: it does not reflect a Pareto efficient use of a unit of income. Rather, it reflects a movement from a Pareto efficient state to a non-Pareto efficient state. That is, to the extent that a unit increase in income causes an increase in tax payments (assuming a positive marginal propensity to pay taxes), the value of λ does not afford any value to these added tax receipts.

Prior to the introduction of a carbon tax, the initial condition of the model reflects a revenue constraint requiring 67% excise taxes on F and N, which is equivalent to a 40% income tax rate. Parameter values for $h(E)$ are chosen so that the marginal social damage from carbon emissions (π/α) is $40. The model is solved for the welfare-maximizing taxes on F and N.

The optimal carbon tax corresponding to $t_F^* - t_N^*$ is $61.3 per ton of carbon, or about 53% higher than marginal social damage (see Table 1 for details). In a first-best setting with no binding revenue-requirements, the optimal carbon tax equals marginal social damage. Hence, we can infer that the optimal

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[2] In the current model the revenue constraint, G, is fixed in nominal rather than real terms.

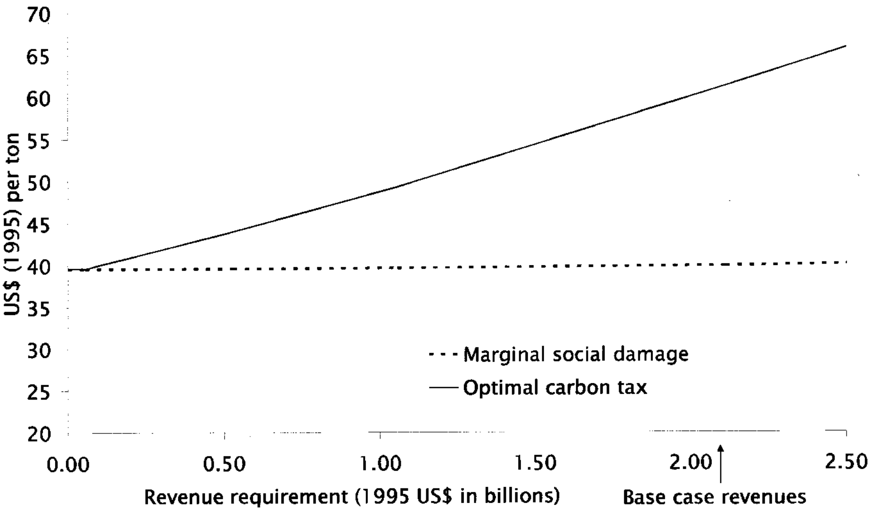

FIGURE 1
RELATIONSHIP BETWEEN THE OPTIMAL CARBON TAX AND MARGINAL SOCIAL DAMAGE IN THE UNITED STATES FOR DIFFERENT LEVELS OF REVENUE REQUIREMENTS

carbon tax rises with increased revenue requirements, as indicated in Figure 1 based on results from the numerical model. For example, increasing the revenue requirement in the numerical model 25% above the initial level produces an optimal carbon tax of $67/ton, or 68% higher than marginal social damages.

The welfare gain from the revenue-neutral introduction of the optimal carbon tax is estimated at $3.58 billion per year. Employment also increases by nearly $900 million in gross wages. These two changes broaden the tax base by introducing a tax on previously untaxed carbon emissions, and by inducing higher labor supply. If the carbon tax is set equal to marginal social damage, however, the welfare gain would be only $3.17 billion and employment would rise by only $640 million.

The intuition for these results can be comprehended in at least three ways. First, the notion underlying the double-dividend hypothesis recognizes that when pollution tax receipts are used to lower revenue-motivated taxes that this will produce a welfare gain by lowering the excess burden of the tax system. This additional benefit from taxing pollution lowers the net cost of emissions reductions, which in turn justifies a higher optimal carbon tax. A second explanation views carbon emissions as simply another good in the economy, one that, like other similar goods, should be taxed above its social cost as part of a broadly based Ramsey tax program. As the revenue requirement increases, so too should the Ramsey tax premiums on all similar goods and services.

A third perspective points out that the global climate can be understood to be an untaxable endowment in much the same way that leisure is not directly taxable. From optimal tax theory we know that revenue-motivated taxes are distortionary precisely because of the existence of untaxed endowments, since individuals are able to avoid taxation by consuming the untaxed endowment. Thus, an increase in revenue-motivated taxes can be expected to result in increased consumption of the untaxable endowments. In the current model with two untaxed endowments, this result is confirmed with increased consumption of "climate quality." In the case of leisure, the increased propensity to consume leisure is offset by a substitution effect as labor productivity rises.

Finally, although these results may at first glance appear to contradict the Samuelson Condition (whereby the optimal provision of a public good is understood to vary inversely with revenue requirements and the cost of public funds), in this case there is no contradiction. The global climate is an endowed public good that is neither directly taxed nor publicly provided, so the cost and expenditure considerations underlying the Samuelson Condition do not directly apply. However, the public good of atmospheric ''waste disposal services'' for greenhouse gases *can* be directly taxed in this case: households can be charged for emissions. Indeed, the model results demonstrate that a rise in the revenue requirement is accompanied by an increase in the carbon tax and a reduction in the consumption of these ''carbon disposal services,'' a change that is fully consistent with the Samuelson Condition for this public good. Amelioration of climate damages comes indirectly, as a byproduct of raising taxes on carbon disposal services as well as on other taxable goods.

IV. THE ''TAX INTERACTION'' LITERATURE

The analyses contained in the tax interaction literature differ from the current analysis in three respects. First, they have generally modeled environmental externalities as amenities that affect utility directly rather than as production externalities that affect output. For a given value of marginal social damage, however, there is no obvious reason why the type of externality being modeled should affect the theoretical relationship between the optimal environmental tax and marginal social damage.

Second, the tax interaction literature uses a tax rule that includes a tax on labor supply (income) rather than by introducing all taxes on expenditures. For this ''income tax normalization,'' N becomes the untaxed good (or intermediate input) rather than leisure, and the tax on labor income serves as the revenue-motivated tax. It is understood in these models that a labor tax is equivalent to uniform taxes on all expenditures, and thus it can serve as an optimal revenue-raising tax if demands are symmetrical, that is if all goods are average substitutes for leisure (see Bovenberg and de Mooij 1994; Bovenberg and Goulder 1996). This symmetric demand condition has often been satisfied by assuming that all goods are homothetic and separable from leisure and environmental quality in the utility function. Like Parry, Williams, and Goulder (1999), the current model satisfies this condition on demands for F and N by assuming that both $c(F, N)$ and $u(C, V)$ are homothetic.

Given this assumption of symmetric demands, a labor tax will represent an optimal revenue raising tax so that the entire remaining tax imposed directly on the polluting good can be interpreted as the environmental tax. This is a convenient attribute for the income tax normalization, however it also may cause confusion because the units in which income is measured will vary with any change in the income tax rate—while at the same time, the unit for government revenues remains unchanged. Indeed, having revenues measured in gross income units and household income measured in net income units may create confusion for interpreting the relationship between the marginal utility of income and the marginal cost of public funds.

In order to minimize the sources of potential confusion on this topic, the current model employs the standard normalization used in the optimal tax literature, where taxes are introduced on expenditures only. Although this introduces a difference between the present model and the models in the ''tax interaction'' literature, it is a difference that will have no effect on the results of the model or on any real variables, and thus should not affect their interpretation (Schöb 1996; Fullerton 1997). Indeed, models and results employing either of these tax normalizations can be easily renormalized, or converted, into the other normalization: it is a matter of redefining units. The current model can be renormalized to reflect a labor tax simply by dividing the household budget constraint in [3] by $1 - t_L$, where t_L is the tax on labor supply and where $1 + t_N = 1/(1 - t_L)$. Whether the numeraire unit of income is defined in gross or net units will not affect the analysis so long as it is used consistently throughout.

The third and pivotal difference between the current analysis and the tax interaction literature is the definition of the marginal social damage from pollution. In the current analysis, the definition of MSD reflects shadow prices, and has been derived based on the marginal rate of substitution between income and environmental quality as expressed in [6]. By contrast, the definition of environmental damage used in the tax interaction literature takes the marginal damage for a representative household, and then sums these across all households. Thus, their definition reflects "marginal private damages" (MPD), or the aggregate marginal willingness to pay to avoid climate damage. The differences between these two definitions have crucial and direct bearing on these results and their interpretation.

In a model with preexisting taxes and a revenue requirement, the social marginal utility of a given resource (either income or environmental quality) will differ from its private marginal utility to the extent that there are taxes present and revenue consequences. Diamond (1985) defined the social marginal utility of income as the gain in social welfare from provision of additional income in numeraire units, "which is the sum of gains from individual consumption and from the marginal propensity to pay taxes out of income" (p. 336). As a result, the social marginal utility of income will exceed the private marginal utility of income so long as the marginal propensity to pay taxes is positive. In the current model, the social marginal utility of a change in climate similarly has two components, one reflecting the changes in private consumption and the other reflecting the changes in tax receipts.

In contrast to this definition, the tax interaction literature has defined the marginal utility of income as λ, the Lagrangian multiplier for the household budget constraint. This reflects only the private consumption component of the social marginal utility for a unit increase in income, and omits the second and third terms in [5] which reflect the utility from the revenues generated by incremental tax payments, and the environmental consequences of changes in consumption corresponding to an increase in income. The difference between the social and private marginal utility of income can be understood as being similar to the distinction in benefit-cost analysis between market prices and shadow prices.

In the "tax interaction" literature, an environmental amenity is assumed to be additively separable in the utility function so that a change in environmental quality will not affect households' allocation choices, and therefore it will have no effect on tax payments or revenues. Thus, due to the absence of any interaction between changes in environmental quality and changes in revenue payments, the social marginal utility of environmental quality (in utility units) will be the same as the private marginal utility of environmental quality (when summed across households).

In the current model, however, climate change damages affect income directly by lowering labor productivity, which reduces labor income and causes households to modify their consumption choices. As a result, the social marginal utility of a change in climate will differ from the private marginal utility of a change in climate due to its revenue consequences. Because climate damages reduce income directly, the difference between the social and private marginal utility of climate will be similar (but not identical) to the differences between the social and private marginal utility of income. Thus, for the current model MPD will differ in both its numerator and denominator from MSD. Let π^P denote the private marginal utility of environmental quality. We can write MPD as

$$\mathrm{MPD} \equiv \frac{\pi^P}{\lambda} m\phi = \frac{m\lambda \dfrac{dh}{dE} L}{\lambda} m\phi. \qquad [8]$$

The numerator and denominator in [8] will be smaller in magnitude than in [6] given the explicit expressions in [4] and [5], so long as the environmental term in [5] is small relative to the revenue term. Private agents will ignore the value of additional revenues generated when their expenditures change as a result of a change in income, and this will be true for exogenous income changes as well as those due to climate change. Because both the numerator and denominator are higher in MSD compared to MPD, the ratios may not differ greatly. Indeed, for the current numerical model where MSD is \$40, MPD is \$35.4. Since the optimal carbon tax is found to be \$61.3, this result implies that, had Parry, Williams, and Goulder (1999) evaluated a model with production externalities, they would have found the optimal carbon tax to be approximately 73% higher than MPD.

It is essential to demonstrate that these numerical results, and the reasons they differ from results in the tax interaction literature, can be reconciled using expressions that have been derived theoretically. The distinction between MSD and MPD is crucial, and the implications of using MPD versus MSD as a benchmark comparator for the optimal carbon tax will be demonstrated using the analytical expressions from the tax interaction literature. The derivation of the analytical expression for the optimal environmental tax for a generalized version of the current model is presented in Appendix B.

An expression for the optimal environmental tax is derived in Bovenberg and Goulder (1996) for their model with an amenity externality (an equivalent expression can be derived from the optimal tax expressions in Sandmo's (1975) seminal article). The expression is

$$\tilde{t}_F^* = \text{MPD}\frac{\tilde{\lambda}}{\mu} = \left(m\phi\frac{\pi}{\tilde{\lambda}}\right)\frac{\tilde{\lambda}}{\mu}, \qquad [9]$$

where a tilde over a variable denotes "net income" units corresponding to the income tax normalization. For their model in which environmental quality is separable in utility so that a change in the amenity externality has no affect on consumption, there will be no difference between the private marginal utility of environmental quality and the social marginal utility of environmental quality. The expression is correct since $\pi = \pi^P$.

The correspondence between the two normalizations implies that $\lambda = \tilde{\lambda}(1 - t_L)$ and $\tilde{t}_F^* = (t_F^* - t_N^*)\,(1 - t_L)$. With these identities, and noting that $\alpha/\alpha = \tilde{\lambda}/\tilde{\lambda}$, we can substitute these expressions into [9] and plug in numerical values from Table 1 to get

$$\begin{aligned} t_F^* - t_N^* &= \frac{\alpha(1 + t_N)}{\mu}\left(m\phi\frac{\pi}{\alpha}\right) \\ &= \frac{0.4(1.641)}{.429}\left(m\phi\frac{\pi}{\alpha}\right) \\ &= 1.53\left(m\phi\frac{\pi}{\alpha}\right). \end{aligned} \qquad [10]$$

Thus, the optimal carbon tax obtained directly from the numerical estimation is shown here to be consistent with one calculated by plugging the shadow values α and μ into the analytical expression. It also confirms that this relationship is independent of the tax normalization.

We can also use the expression in [9] to point out those circumstances under which the optimal environmental tax may appear to be less than marginal damages. Consider a model like those used in the tax interaction literature involving an environmental amenity that is separable in utility, and where marginal environmental damage is defined by MPD rather than MSD. As explained above, these restrictions on preferences ensure that a change in environmental quality will not affect households' allocation choices or tax payments, so that the social and private marginal utility of environmental quality will be equal ($\pi = \pi^P$). Thus, the numerators in MSD and MPD will be identical. The only difference in this case between MPD and MSD will be due to having λ in the denominator of MPD rather than α which is in the denominator of MSD. We can express [9] by substituting $\lambda(1 + t_N) = \tilde{\lambda}$ and plugging in values from Table 1 to get

$$\begin{aligned} \tilde{t}_F^* &= \frac{\lambda(1 + t_N)}{\mu}\left(m\phi\frac{\pi}{\tilde{\lambda}}\right) \\ &= \frac{0.255(1.641)}{0.932}\left(m\phi\frac{\pi}{\tilde{\lambda}}\right) \\ &= 0.97\left(m\phi\frac{\pi}{\tilde{\lambda}}\right). \end{aligned} \qquad [11]$$

When defining marginal environmental damage using λ rather than α in the denominator, the relationship between the optimal environmental tax and the benchmark (MPD in this case) is altered. Because the numerators for both MPD and MSD are the same, but λ is less than α, it is possible that MPD will exceed the optimal environmental tax, even while MSD is less than the optimal environmental tax.[3]

This particular result, however, should not cast doubt on the validity of the double dividend hypothesis. Comparisons between MPD and the optimal environmental tax may appear to be incongruous with our intuition, but this is because MPD does not reflect the social value of a numeraire unit of income: λ does not reflect the value of a unit of income allocated to a Pareto efficient use. For optimization models such as the one represented by [3] where households and planners are understood to respond to each other's actions, λ only reflects the increased consumption by households from a unit increase in income, but with no response or change on the part of the social planner to the changes in tax receipts. For an increase in income, this implies that the incremental revenues generated have no value. Indeed, for a decrement of income, λ would appear to reflect a change that is infeasible: a unit decrease in income will lower consumption and tax payments, but transfers of G (or the provision of public goods) remain unchanged.

Thus, in the case of an environmental amenity, MPD may exceed the optimal environmental tax making it appear as though the optimal tax lies below the ''the Pigouvian rate that fully internalizes the marginal environmental damages'' (Bovenberg and de Mooij 1994). But this interpretation is misleading. Had the authors of the tax interaction literature used MSD rather than MPD as their benchmark comparator, they would have found that the optimal environmental tax exceeded that level. Alternatively, had they modeled a production externality rather than an amenity externality, they also would have found that the optimal environmental tax exceeded marginal environmental damage (no matter whether they used MPD or MSD). Only for the particular case of an amenity externality, and when using MPD as the measure of marginal damages, does the optimal environmental tax generally appear to be lower than marginal damages in these models.[4]

IV. CONCLUDING COMMENTS

The welfare effects of environmental tax reform such as the introduction of a carbon tax have become an important policy question as well as a source of debate in the theoretical literature. It is accepted wisdom that using carbon tax revenues to finance reductions in preexisting taxes will lower the excess burden of the tax program. Intuitively this implies that the net welfare change from introducing a carbon tax will exceed that which would be inferred by simply comparing marginal social damages to marginal abatement costs; and it follows logically that the optimal carbon tax will exceed the marginal social damage (Lee and Misiolek 1986; Terkla 1984). The current analysis is consistent with this intuition and the double dividend hypothesis. In a second-best setting with preexisting taxes, and where carbon damages affect production, the optimal carbon tax is found to exceed marginal social damages by 53% based on a numerical model of the U.S. economy. When marginal climate damages are defined or estimated as the sum of household's marginal willingness to pay, then the optimal carbon tax will exceed this measure of marginal damages by 73%. The welfare gain from the revenue-neutral introduction of the optimal carbon tax when marginal social damage is $40 per ton is estimated at $3.57 billion per year for the U.S. economy. Employment also increases by $900 million in gross wages annually.

Differences between the current findings

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[3] For the current numerical model, $\pi/\alpha = 40$, $\pi^P/\lambda = 35.4$, and $\alpha/\lambda = 1.57$, from which we can infer that $\pi/\pi^P = 1.77$. This ratio is larger than α/λ due to the substitution effect of a change in labor productivity, whereas α is strictly an income effect.

[4] Of course, these generalizations can only be assumed to hold under the standard assumptions that have been made in this literature: that labor supply is upward sloping and that all goods are average substitutes for leisure.

and those in the tax interaction literature are shown to stem from differences in the way marginal social damage has been defined. In the current analysis, marginal social damage is defined based on the marginal rate of substitution between income and environmental quality from society's perspective. In the tax interaction literature, marginal damage has been defined in a way that does not reflect the marginal rate of substitution between income and environmental quality; their definition is an aggregation across households of marginal private damages, reflecting the private marginal utility of income rather than the social marginal utility of income.

The essential point for understanding the source of these ostensibly contradictory results is that as revenue-motivated tax rates rise, the private marginal utility of income will decline relative to the social marginal utility of income, and the divergence between the two definitions will rise with increasing revenue requirements. If these distinctions are overlooked, the expected and intuitive correspondence between the optimal environmental tax, marginal social damage and the validity of the double dividend hypothesis will become obscured.

Defining the social value of a resource without taking account of distortionary taxes or subsidies is contrary to well-established concepts and methods of shadow pricing. This point should not be controversial. Consider for example if climate change damages were being valued based on sea level rise and the consequent losses of agricultural lands. Economists would be quick to point out that market prices for farmland should not be used to measure these damages if farm subsidies have inflated their values. Obviously, the shadow value of agricultural land is the appropriate measure rather than the private value. The same logic applies to valuing other resources such as income and environmental quality.

Given the need to provide policymakers with clear guidance on climate change policy questions, these results, and the reasons for the ostensibly incongruous findings in the tax interaction literature, need to be fully recognized and understood in order to avoid confusion by both policymakers and the general public. For example, using a model typical of the tax interaction literature, Bovenberg and van der Ploeg (1994) concluded that expanding environmental policy will tend to slow the economy and lower employment, forcing governments to choose between a clean environment and the provision of other public goods. Evidence suggests that these interpretations have been influential in public and political debates on environmental policies such as green tax reform in Europe and the United States. The current analysis concludes that environmental policy can actually increase employment and economic activity, and that environmental taxation lowers, rather than raises, the cost of raising revenues to achieve other collective goals.

APPENDICES

Appendix A: Specification of the Numerical Climate-Economy Model

The numerical model representing the U.S. economy is represented in [7]. The functional forms and parameter values are described here. These include a primary CES utility function, $U = u(C, V)$ given as

$$U = [\gamma C^{-\rho} + (1 - \gamma)V^{-\rho}]^{-1/\rho},$$

and a secondary CES production function defining substitutions between F and N in $C = c(F, N)$ as

$$C = [\beta F^{-\delta} + (1 - \beta)N^{-\delta}]^{-1/\delta}.$$

This production function is a single CES function rather than the more disaggregated, nested CES structure of production in the Parry, Williams, and Goulder model (1999). The function $H = h(E)$ is $H = 1 - 0.00001122(E - 1423.6)$, and $\phi = 0.00225113$. The functions and parameters have been calibrated to correspond to the second-best marginal abatement cost function from Parry, Williams, and Goulder (1999). Setting $\delta = -0.5$ implies that the elasticity of substitution between carbon emitting and non-carbon emitting consumption, σ_{NF}, equals $(1/1 + \delta) = 2.0$. The value of $\rho = -0.167$, so that the elasticity of substitution between consumption and leisure, σ_{CV}, equals $(1/1 + \rho) = 1.2$. In addition, $\gamma = 0.836$, $\beta = 0.667$, and $m = 1$.

The desired labor supply elasticity (0.15) is achieved by calibrating the share parameter γ. The initial conditions are $T = 4101534.9$, $G = 2{,}113{,}100$, $V = 931{,}980$ and $E = 1423.6$. These are identical to the model in Parry, Williams, and Goulder (1999).

Appendix B: Derivation of the Optimal Environmental Tax for a Production Externality

In the interest of notational convenience, and to generalize for many goods, detailed derivations for a general model with n goods are presented, where X_z is the notational equivalent to F. The essential problem can be stated as one in which m identical individuals maximize utility $U = u(X_0, X_1, \ldots, X_Z, \ldots, X_n)$ for goods $j = 0, \ldots, n$, where leisure is X_0 and where labor supply is taken out of a time endowment normalized to equal one so that labor supply equals $1 - X_0$. Units are chosen for goods and income so that all pre-tax prices equal one, and where there are $n - 1$ non-polluting X goods (excluding leisure) and one good, X_z, which produces an environmental externality. For a production externality we assume that labor productivity, h, is a function of environmental quality such that $h = h(E)$. We define aggregate output as $mh(E)L = \sum mX_i$, and where mG is financed through collection of tax revenues. Each household's maximization problem can be stated as

$$\underset{X_0 \ldots X_n}{\text{Max:}} \quad u(X_0, X_1, \ldots X_n)$$

$$s.t. \qquad hL + G = \sum_{j=1}^{n} (1 + t_j)X_j, \qquad [A1]$$

so that individuals maximize utility subject to their budget constraint while ignoring both the environmental consequences of their own consumption choices and government behavior. The Lagrangian expression for each household is thus

$$\mathcal{L} = u(X_0, X_1, \ldots X_n) + \lambda\left[h(1 - X_0) + G - \sum_{j=1}^{n} (1 + t_j)X_j\right]$$

$$\text{for } j = 1, \ldots, z, \ldots, n. \qquad [A2]$$

Consumer prices are given as $p_j = 1 + t_j$, for $j = 1$ to n, but where income is untaxed, so that $p_0 = 1$. The first-order conditions for each household take the form $U_j = \lambda(1 + t_j)$, for $j = 1, \ldots, n$, and $U_o = \lambda h(E)$, for X_0.

The social planner's problem is then

$$\underset{t_l \ldots t_\mathrm{n}}{\text{Max:}}$$

$$m\left[u(X_0, X_1, \ldots X_n)\, s.t.\, h(1 - X_0) + G = \sum_{n=1}^{n} (1 + t_j)X_j\right]$$

$$s.t. \qquad m\sum_{j=1}^{n} t_jX_j = G$$

$$h = h(E(mX_Z)). \qquad [A3]$$

Taking the dual approach, we define the household's indirect utility function as $v(p_0, p_1, \ldots p_n) = u(x(p_0, p_1, \ldots p_n))$, so we can state the social optimization problem as the Lagrangian equation

$$\mathcal{L} = mu(p_0, p_1, \ldots p_n) + \mu\left[m\sum_{j=1}^{n} t_jX_j - G\right].$$

In the presence of environmental effects on labor productivity, the first-order conditions for the social optimization problem are

$$-\lambda X_j + \mu\left[\sum_{j=1}^{n} t_i\frac{\partial X_i}{\partial p_j} + X_j\right]$$
$$+ \left[\lambda\left(m(1 - X_0)\frac{\partial h}{\partial E}\frac{de}{d(mX_Z)}\right)\right.$$
$$\left. + \mu\sum_{j=1}^{n} t_i\frac{\partial X_i}{\partial(h(1 - X_0))}\frac{\partial(h(1 - X_0))}{\partial E}\frac{de}{d(mX_Z)}\right]$$
$$\times \frac{\partial X_Z}{\partial p_j} = 0 \quad \forall j \neq 0, \qquad [A4]$$

where $\lambda = dv/d(h(1 - X_0)) = \partial U^*/\partial(h(1 - X_0))$ is the household's marginal utility of income. Let π denote marginal social damages in utility units, or

$$\pi = \lambda\left(mL\frac{\partial h}{\partial E}\frac{de}{d(mX_z)}\right)$$
$$+ \mu\sum_{j=1}^{n} t_i\frac{\partial X_i}{\partial(h(1 - X_0))}\frac{\partial(h(1 - X_0))}{\partial E}\frac{de}{d(mX_Z)}, \qquad [A5]$$

where marginal social damage includes both the direct loss of income to households as well as the loss in revenues due to changes in labor income, $h(1 - X_0)$, due to the change in h as well as the resulting change in labor supply.

Simplifying the notation in [A4] using [A5] we have

$$-\lambda X_j + \mu\left[\sum_{j=1}^{n} t_i \frac{\partial X_i}{\partial p_j} + X_j\right] + \pi \frac{\partial X_Z}{\partial p_j} = 0 \quad \forall\, j \neq 0. \qquad \text{[A6]}$$

Derivations of optimal tax rules often include substitution of the Slutsky equation in such a way that the social marginal utility of income, α, is represented along with the shadow cost of raising an additional dollar of revenue (Auerbach 1985). Diverging from the approach taken by Sandmo (1975), we rearrange the planner's first-order conditions and use the Slutsky equation to split the cross-price effects into compensated effects (superscript U) and effects on income, Y, as

$$\frac{\partial X_Z}{\partial p_i} = \frac{\partial X_Z^U}{\partial p_i} - X_i \frac{\partial X_Z}{\partial Y}.$$

We substitute α to obtain

$$-\alpha X_j + \mu \sum_{i}^{n} t_i \frac{\partial X_i^U}{\partial p_j} + \mu X_j + \pi\left(\frac{\partial X_Z^U}{\partial p_j} - X_j \frac{\partial X_Z}{\partial Y}\right) = 0 \quad \forall\, j \neq 0. \qquad \text{[A7]}$$

We define Š as the determinant of the Slutsky matrix of compensated demands, so that S_{ij} is the cofactor of the element for the *jth* row (price) and *ith* column (quantity). Using Cramer's rule we can solve for the optimal taxes

$$t_j = \frac{(\mu - \alpha)}{\mu} \frac{\sum_{i=1}^{n} X_i S_{ij}}{\hat{S}} + \frac{\pi}{\mu} \frac{\sum_{i=1}^{n} \left(\frac{\partial X_Z^U}{\partial p_i} - X_i \frac{\partial X_Z}{\partial Y}\right) S_{ij}}{\check{S}}, \qquad \text{[A8]}$$

where the second term on the right-hand side is the environmental component of the tax. From theorems about the expansion of determinants, we know that

$$\sum_{i=1}^{n} \frac{\partial X_Z^U}{\partial p_i} S_{ij} = \begin{cases} 0 & \text{for } j \neq Z \\ \check{S} & \text{for } j = Z. \end{cases}$$

Let R denote the "Ramsey term" for compensated demands or $R \equiv \sum_{i=1}^{n} X_i S_{ij} / p_j \check{S}$ reflecting the revenue generating potential for a marginal change in the tax on X_i due to the direct and indirect effects on consumption for all goods. Further simplify the notation by defining the income effect on the environment as

$$\theta = \pi \sum_{i=1}^{n} X_i \frac{\partial X_Z}{\partial Y}.$$

We can thus rearrange terms and simplify so that the optimal tax expressions can then be written as

$$\frac{t_j}{(1 + t_j)} = \frac{(\mu - \alpha + \theta)}{\mu} R \quad \forall\, j \neq z. \qquad \text{[A9]}$$

and

$$\frac{t_j}{(1 + t_j)} = \frac{(\mu - \alpha + \theta)}{\mu} R + \frac{\pi}{\mu(1 + t_j)} \quad \forall\, j = z. \qquad \text{[A10]}$$

These implicit solutions are difficult to interpret by inspection, in part because of the lack of transparency in interpreting R. Moreover, although the environmental component of the tax in [A10] appears to be separable from the standard formula, the independence is illusory both because the denominator $(1 + t_z)$ is endogenous and because the actual level of the externality depends on the actual equilibrium and hence the optimal tax rates.

The results differ from the expressions obtained by Sandmo involving uncompensated demands. Sandmo concluded that the environmental damages of X_Z "does not enter the tax formulas for the other commodities, regardless of the pattern of complementarity and substitutability" (Sandmo 1975, 92). In this alternative derivation, we see that the numerator in the first term on the right-hand side includes θ, a term involving π, indicating that the presence of an externality raises the optimal tax on all goods due to their income effect: by reducing real income, all taxes discourage consumption of the externality-producing good to some extent, and these optimal tax rates will be higher as a result. These two versions of the optimal tax results are not in conflict: in the model involving ordinary demands, the income effects are implicit.

The relationship between the optimal environmental tax and marginal environmental damages can be evaluated based on the optimal tax expressions derived above. Noting the identity MSD = π/α and reverting to the notation where F is the polluting good and N is the non-polluting good, we can write these as

$$\frac{t_N^*}{(1 + t_N^*)} = \left(\frac{\mu - \alpha + \theta}{\mu}\right)R \qquad [A11]$$

and

$$\frac{t_F^*}{(1 + t_F^*)} = \left(\frac{\mu - \alpha + \theta}{\mu}\right)R + \frac{\alpha}{\mu}\frac{\text{MSD}}{(1 + t_F)}, \qquad [A12]$$

where θ represents the direct and indirect income effects on the environment.

These expressions, similar to those derived by Sandmo, are difficult to interpret by inspection because of the lack of transparency in R, and the environmental component of the optimal tax for F cannot be evaluated separately by inspection since the left-hand side denominator is a function of both terms.[5] The problem, however, can be simplified by assuming that all goods are similar from a revenue raising perspective. This has also been done in the recent literature by restricting preferences so that utility is homothetic in consumption, and weakly separable in leisure, environmental quality, and government consumption (Bovenberg and de Mooij 1994; Bovenberg and Goulder 1996). Here, however, we assume only that the Ramsey terms for both goods are equal, which allows us to derive an expression for the environmental component of the optimal tax.

For the polluting good, F, we rearrange [A12] to get

$$t_F^* = \frac{\left(1 - \frac{\alpha + \theta}{\mu}\right)R}{\left(1 - \left(1 - \frac{\alpha + \theta}{\mu}\right)R\right)} + \frac{\alpha\text{MSD}}{\mu\left(1 - \left(1 - \frac{\alpha + \theta}{\mu}\right)R\right)}. \qquad [A13]$$

From [A11] we can express the Ramsey term as

$$R = \frac{\frac{t_N^*}{(1 + t_N^*)}}{\left(1 - \frac{\alpha + \theta}{\mu}\right)}. \qquad [A14]$$

To evaluate the optimal tax t_F^* relative to MSD, we substitute [A14] into the second term of [A13] and rearrange to get

$$t_F^* = \frac{\left(1 - \frac{\alpha + \theta}{\mu}\right)R}{\left(1 - \left(1 - \frac{\alpha + \theta}{\mu}\right)R\right)} + \frac{\alpha(1 + t_N)\text{MSD}}{\mu}. \qquad [A15]$$

We can evaluate the relationship between the optimal environmental tax and MSD in one of two ways; either evaluating the difference between the optimal tax expressions for N and F or by differentiating t_F with respect to MSD. Taking the differentiation approach, we obtain

$$\frac{\partial t_F^*}{\partial(\text{MSD})} = \frac{(1 + t_N)\alpha}{\mu}, \qquad [A16]$$

which indicates that the optimal tax may rise by more or less than MSD depending on the relationship between the revenue-motivated tax rate t_N^* and α/μ.

To demonstrate the similarity between these results and those found in Bovenberg and Goulder (1996), note that an income tax normalization amounts to multiplying the household budget constraint by $(1 - t_L)$ where t_L is the income tax rate and where $(1 + t_N) = 1/(1 - t_L)$. This step implies that the household budget constraint now represents units of net income. Let a tilde (~) denote net income units, so that the shadow value of net income is $\tilde{\lambda}$, which will differ from the shadow value of gross income, λ, such that $\lambda =$

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[5] Although Fullerton (1997), Schöb (1997), and Bovenberg and de Mooij (1997) have suggested that Sandmo's formula indicates that the optimal pollution tax should be less than marginal social damages, this interpretation overlooks the endogeneity of the denominator on the left-hand side: one cannot infer by inspection that the differential between the optimal tax on a polluting good versus a similar non-polluting good will simply equal the value of the second term (since the first term equals the tax on non-polluting goods).

$\tilde{\lambda}(1 - t_L)$. In the Bovenberg and Goulder model, household income is expressed in units of net income whereas the government revenue constraint continues to be expressed in units of gross income (as in [3]), which means that the shadow value on their revenue constraint, μ, reflects gross income units. To be consistent, this shadow value can also be expressed in net income units by substituting the identity $\mu = \tilde{\mu}(1 - t_L)$.

If we also substitute the identity, $\tilde{\lambda}/\tilde{\lambda} = \tilde{\alpha}/\tilde{\alpha}$, Bovenberg and Goulder's result in [9] for an amenity externality can be expressed as

$$\tilde{t}_F^* = \left(m\phi\frac{\pi}{\tilde{\alpha}}\right)\frac{\tilde{\alpha}}{\tilde{\mu}(1 - t_L)}. \qquad [A17]$$

We can differentiate [A17] with respect to MSD ($= m\phi\pi/\tilde{\alpha}$) to get

$$\frac{\partial\tilde{t}_F^*}{\partial(\mathrm{MS\tilde{D}})} = \frac{\tilde{\alpha}}{\tilde{\mu}(1 - t_L)} = \frac{(1 + t_N)\tilde{\alpha}}{\tilde{\mu}}. \qquad [A18]$$

This expression is identical to [A16] but for the difference in normalization.

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